Eur. Phys. J. C 28, 557–560 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01185-5

THE EUROPEAN PHYSICAL JOURNAL C

Strong and electroweak interactions and their unification with non-commutative space-time

X.-G. He[1,2]

[1] Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China
[2] Department of Physics, National Taiwan University, Taipei 10764, Taiwan

Received: 18 September 2002 / Revised version: 21 February 2003 /
Published online: 5 May 2003 –

Abstract. Quantum field theories based on non-commutative space-time (NCQFT) have been extensively studied recently. However no NCQFT model which can uniquely describe the strong and electroweak interactions has been constructed. This prevents one to make a consistent and systematic study of non-commutative space-time. In this work we construct a NCQFT model based on the trinification gauge group $SU(3)_C \times SU(3)_\mathrm{L} \times SU(3)_\mathrm{R}$. A unique feature of this model, that all matter fields (fermions and Higgs bosons) are assigned to (anti-) fundamental representations of the factor $SU(3)$ groups, allows us to construct a NCQFT model for strong and electroweak interactions and their unification without ambiguities. This model provides an example which allows one to make a consistent and systematic study of non-commutative space-time phenomenology. We also comment on some related issues regarding extensions to E_6 and $U(3)_C \times U(3)_\mathrm{L} \times U(3)_\mathrm{R}$ models.

Non-commutative quantum field theory (NCQFT), based on a modification of the space-time commutation relations, provides an alternative to ordinary quantum field theory. A simple way to modify the space-time properties is to change the usual space-time coordinate x to a non-commutative coordinate $\hat{X}$ such that [1]

$$[\hat{X}^\mu, \hat{X}^\nu] = \mathrm{i}\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu}$ is a real anti-symmetric matrix. We will consider the case where $\theta^{\mu\nu}$ is a constant and commutes with $\hat{X}^\mu$. NCQFT based on the above commutation relation can easily be studied using the Weyl–Moyal correspondence replacing the product of two fields $A(\hat{X})$ and $B(\hat{X})$ with non-commutative coordinates by a product of the same fields but an ordinary coordinate x through the star "$*$" product,

$$\begin{aligned} A(\hat{X})B(\hat{X}) &\to A(x) * B(x) \\ &= \mathrm{Exp}\left[\mathrm{i}\frac{1}{2}\theta^{\mu\nu}\partial_{x,\mu}\partial_{y,\nu}\right] A(x)B(y)|_{x=y}. \end{aligned} \tag{2}$$

Properties related to NCQFT have been studied extensively recently [2–11,13,14]. NCQFT for a pure $U(1)$ group is easy to study. The related phenomenology has been studied recently [2]. But it is more complicated for non-abelian groups. Due to the nature of the "$*$" product, there are fundamental differences between ordinary and non-commutative gauge theories and these cause many difficulties in the construction of a unique and consistent model for the strong and electroweak interactions based on the $SU(3)_C \times SU(2)_\mathrm{L} \times U(1)_Y$ gauge group [3–11,13,14].

One of the main problems is that the $SU(N)$ group cannot be simply gauged with the "$*$" product as will be explained in the following. Another problem is that, naively, the charges of any $U(1)$ gauge group with the "$*$" product are quantized to only three possible values, 1, 0, −1, which cannot accommodate all the hypercharges for matter fields in the SM.

In this work we construct a NCQFT model based on the trinification gauge group $SU(3)_C \times SU(3)_\mathrm{L} \times SU(3)_\mathrm{R}$. We show that the NCQFT model for the strong and electroweak interactions and their unification can be consistently constructed. This model therefore provides an example which allows one to make a consistent and systematic study of the non-commutative space-time phenomenology. With a non-commutative space-time there are modifications to the fields compared with the ordinary ones. We indicate the fields in NCQFT with a hat and the ordinary ones without a hat. The definition of the gauge transformation $\hat{\alpha}$ of a gauge field $\hat{A}_\mu$ for $SU(N)$ is similar to the ordinary one, but with the usual product replaced by the "$*$" product. For example

$$\delta_\alpha \hat{\phi} = \mathrm{i}\hat{\alpha} * \hat{\phi}, \tag{3}$$

where $\hat{\phi}$ is a fundamental representation of $SU(N)$. We use the notation $\hat{A}_\mu = \hat{A}^a_\mu T^a$, $\hat{\alpha} = \alpha^a T^a$ with T^a being the $SU(N)$ generator normalized by $\mathrm{Tr}(T^aT^b) = \delta^{ab}/2$. Due to the non-commutativity of the space-time, two consecutive local transformations $\hat{\alpha}$ and $\hat{\beta}$ of the type above,

$$(\delta_\alpha\delta_\beta - \delta_\beta\delta_\alpha) = (\hat{\alpha} * \hat{\beta} - \hat{\beta} * \hat{\alpha}), \quad (4)$$

cannot be reduced to the matrix commutator of the generators of the Lie algebra due to the non-commutativity of the space-time. They have to be in the enveloping algebra

$$\hat{\alpha} = \alpha + \alpha^1_{ab} : T^a T^b : +... + \alpha^{n-1}_{a_1...a_n} : T^{a_1}...T^{a_n} : +... \quad (5)$$

where $: T^{a_1}...T^{a_n} :$ is totally symmetric under exchange of the a_i. This poses a difficulty in constructing non-abelian $SU(N)$ gauge theories [3]. Seiberg and Witten have shown [5] that the fields defined in non-commutative coordinates can be mapped on to the ordinary fields, the Seiberg–Witten mapping. In [6] it was shown that this mapping actually can be applied to the "$*$" product with any gauge groups. It is possible to study non-abelian gauge group theories. Using the above enveloping algebra, one can obtain the non-commutative fields in terms of the ordinary fields with corrections in powers of the non-commutative parameter, $\theta^{\mu\nu}$, order by order. To the first order in $\theta^{\mu\nu}$, non-commutative fields can be expressed by

$$\begin{aligned}
\hat{\alpha} &= \alpha + \frac{1}{4}\theta^{\mu\nu}\{\partial_\mu\alpha, A_\nu\} + c\theta^{\mu\nu}[\partial_\mu\alpha, A_\nu], \\
A_\mu &= -\frac{1}{4}\theta^{\alpha\beta}\{A_\alpha, \partial_\beta A_\mu + F_{\beta\mu}\} \\
&\quad + c\theta^{\alpha\beta}([A_\alpha, \partial_\mu A_\beta] + \mathrm{i}[A_\alpha A_\beta, A_\mu]), \quad (6)\\
\hat{\phi} &= a\theta^{\mu\nu}F_{\mu\nu}\phi - \frac{1}{2}\theta^{\mu\nu}A_\mu\partial_\nu\phi + \mathrm{i}\left(\frac{1}{4}+c\right)\theta^{\mu\nu}A_\mu A_\nu\phi,
\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g_N[A_\mu, A_\nu]$. The term proportional to a can be absorbed into the redefinition of the matter field ϕ. The parameter c cannot be removed by a redefinition of the gauge field. It must be a purely imaginary number from the requirement that the gauge field be self-conjugate. Using the above non-commutative fields, one can construct a gauge theory for the $SU(N)$ group. The action S of a $SU(N)$ NCQFT, to the leading order in θ, is given by [6,7]

$$\begin{aligned}
S &= \int L\mathrm{d}^4x, \\
L &= -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\
&\quad + \frac{1}{4}g_N\theta^{\mu\nu}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - 4F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}) \\
&\quad + \bar{\phi}(\mathrm{i}\gamma^\mu D_\mu - m)\phi - \frac{1}{4}\theta^{\alpha\beta}\bar{\phi}F_{\alpha\beta}(\mathrm{i}\gamma^\mu D_\mu - m)\phi \\
&\quad - \frac{1}{2}\theta^{\alpha\beta}\bar{\phi}\gamma^\mu F_{\mu\alpha}\mathrm{i}D_\beta\phi, \quad (7)
\end{aligned}$$

where $D_\mu = \partial_\mu - \mathrm{i}g_N T^a A^a_\mu$. We note that the parameter c does not appear in the Lagrangian. The Lagrangian is uniquely determined to order θ. We will therefore work with the simple choice $c = 0$ from now on. In the above, if ϕ is a chiral field, $m = 0$. To obtain a theory which can describe the strong and electroweak interactions such as the standard model (SM), one also needs to solve the $U(1)$ charge quantization problem, namely the existence of only three possible values, 1, 0, -1, for the $U(1)$ charges, as mentioned earlier. It has been shown that this difficulty can also be overcome with the use of the Seiberg–Witten mapping [5]. To solve the $U(1)$ charge quantization problem, one associates with each charge $gq^{(n)}$ of the nth matter field a gauge field $\hat{A}^{(n)}_\mu$ [8]. In the commutative limit, $\theta^{\mu\nu} \to 0$, $\hat{A}^{(n)}_\mu$ becomes the single gauge field A_μ of the ordinary commuting space-time $U(1)$ gauge theory. But at non-zero orders in $\theta^{\mu\nu}$, $\hat{A}^{(n)}_\mu$ receives corrections [8].

In doing so, the kinetic energy of the gauge boson will, however, be affected. Depending on how the kinetic energy is defined (weight over different field strengths of $\hat{A}^{(n)}_\mu$), the resulting kinetic energy will be different, even though the proper normalization to obtain the correct kinetic energy in the commutative limit is imposed [8]. In the SM there are six different matter field multiplets for each generation, i.e. u_R, d_R, $(u,\ d)_\mathrm{L}$, e_R, $(\nu,\ e)_\mathrm{L}$ and $(H^0,\ H^-)$; a priori one can choose a different g_i for each of them. After identifying the three combinations with the usual g_3, g_2 and g_1 couplings for the SM gauge groups, there is still a freedom to choose different gauge boson self-interaction couplings at non-zero orders in $\theta^{\mu\nu}$. This leads to ambiguities in the self-interactions of the gauge bosons when non-zero order terms in $\theta^{\mu\nu}$ are included [8]. This problem needs to be resolved. A way to solve this problem is to have a theory without the use of the $U(1)$ factor group. There are many groups without $U(1)$ factor group which contain the SM gauge group and may be used to describe the strong and electroweak interactions. However not all of them can be easily extended to a full NCQFT using the formulation described above. For example one can easily obtain unique gauge boson self-interactions in $SU(5)$ theory [9]. But the matter fields require more than one representations 5 and 10 which causes additional complications [10] and will reintroduce the uniqueness problem for the kinetic energy. One way of obtaining a consistent NCQFT is to have a theory in which all matter and Higgs fields are in the same representation such that once the Seiberg–Witten mapping is used to solve the problem of gauging a $SU(N)$ group, there is no problem with the unique determination of the kinetic energy. To this end we propose to use the trinification gauge group [15], $SU(3)_C \times SU(3)_\mathrm{L} \times SU(3)_\mathrm{R}$, with a Z_3 symmetry. This theory leads to the unification of the strong and electroweak interactions. An important feature of this theory is that the matter and Higgs fields are assigned to (anti-) fundamental representations of the factor $SU(3)$ groups and therefore the formalism described earlier can be readily used. In the trinification model, the gauge fields are in the adjoint representation,

$$24 = A^C + A^\mathrm{L} + A^\mathrm{R} = (8,1,1) + (1,8,1) + (1,1,8), \quad (8)$$

which contains 24 gauge bosons. A^C contains the color gluon bosons, a linear combination of component fields of A^L and A^R forms the $U(1)_Y$ gauge boson, and A^L contains the $SU(2)_\mathrm{L}$ gauge bosons. The rest are integrally charged heavy gauge bosons which do not mediate proton decays [15]. Each generation of fermions is assigned to a 27,

$$\psi = \psi^{\rm LR} + \psi^{{\rm R}C} + \psi^{C{\rm L}} = (1,3,\bar{3}) + (\bar{3},1,3) + (3,\bar{3},1),$$

$$\psi^{\rm LR} = \begin{pmatrix} E^0 & E^- & e^- \\ E^+ & \bar{E}^0 & \nu \\ e^+ & N_1 & N_2 \end{pmatrix}, \quad \psi^{{\rm R}C} = \begin{pmatrix} \bar{u}_1 & \bar{u}_2 & \bar{u}_3 \\ \bar{d}_1 & \bar{d}_2 & \bar{d}_3 \\ \bar{B}_2 & \bar{B}_2 & \bar{B}_2 \end{pmatrix},$$

$$\psi^{C{\rm L}} = \begin{pmatrix} u_1 & d_1 & B_1 \\ u_2 & d_2 & B_2 \\ u_3 & d_3 & B_3 \end{pmatrix}. \tag{9}$$

In the above we have written the fermions in left-handed chiral fields. The B field is a heavy particle. The Higgs fields which break the trinification to the SM gauge group are also assigned to the 27 representations. In order to have correct mass patterns, at least two 27 Higgs representations are needed [15]. We indicate them by

$$\phi_i = \phi_i^{\rm LR} + \phi_i^{{\rm R}C} + \phi_i^{C{\rm L}} = (1,3,\bar{3})_i + (\bar{3},1,3)_i + (3,\bar{3},1)_i. \tag{10}$$

The Z_3 symmetry operates in the following way. If $(C,{\rm L},{\rm R})$ is a representation under $SU(3)_C \times SU(3)_{\rm L} \times SU(3)_{\rm R}$, the effect of Z_3 is to symmetrize it to

$$Z_3(C,{\rm L},{\rm R}) = (C,{\rm L},{\rm R}) + ({\rm R}C{\rm L}) + ({\rm L},{\rm R},C). \tag{11}$$

The requirement of the Lagrangian to be invariant under Z_3 relates the gauge couplings $g^{C,{\rm L},{\rm R}}$ of the gauge groups and makes them equal, $g^C = g^{\rm L} = g^{\rm R} = g^U$, at a scale which is the unification scale of the model. The vacuum expectation values of the Higgs scalars break the symmetry to $SU(3)_C \times SU(2)_{\rm L} \times U(1)_Y$; this leads to the following form:

$$\langle \phi_1^{\rm LR} \rangle = \begin{pmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \quad \langle \phi_2^{\rm LR} \rangle = \begin{pmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & \hat{0} \\ 0 & v_2 & \hat{0} \end{pmatrix}. \tag{12}$$

Non-zero values of $v_{1,2}$ break the symmetry to the SM group. The scale of $v_{1,2}$ are at the unification scale. At this stage 12 of the gauge bosons, and B, E_i and N_i particles receive masses. They are therefore very heavy. The entries indicated by $\hat{0}$ can develop VEVs of order m_W. These VEVs break the SM group to $SU(3)_C \times U(1)_{\rm em}$ and provide masses for the ordinary quarks and leptons. In this model $\sin^2\theta_{\rm W} = 3/8$ at the unification scale. Using the present electroweak precision test data for $\sin^2\theta_{\rm W}$, $\alpha_{\rm s}$ and $\alpha_{\rm em}$ at the Z mass pole, the unification scale is determined to be 10^{14} GeV [16]. A unification scale as low as 10^{14} GeV in a $SU(5)$ theory, for example, would induce rapid proton decays and is ruled out. However, as has been mentioned previously, in the trinification theory gauge bosons do not mediate proton decays. Therefore the theory would not have the problem with proton decays. Mediation of heavy Higgs particles can produce proton decays [15]. However in this case, there are many free parameters in the Yukawa and the Higgs potential couplings to make the theory consistent with the data [15]. It is clear that the trinification model can provide an easy framework for building a phenomenologically acceptable and consistent NCQFT model for the strong and electroweak interactions. To the first order in $\theta^{\mu\nu}$, the non-commutative gauge fields are of the same form for the gauge fields as in (6). The fermion and Higgs fields are in the same representation and are all (anti-) fundamental representations ϕ of the type $(3,\bar{3})$ under the subgroups $SU(3) \times SU(3)$. The non-commutative fields expressed in the ordinary fields are given by

$$\hat{\phi} = \phi - \frac{1}{2}\theta^{\mu\nu} \times \left(A_\mu \partial_\nu \phi - \frac{\rm i}{2} A_\mu A_\nu \phi + \partial_\nu \phi A'_\mu + \frac{\rm i}{2} \phi A'_\nu A'_\mu \right), \tag{13}$$

where A_μ and A'_μ are the gauge fields of the first $SU(3)$ and the second $SU(3)$ gauge groups, respectively. The corrections to the Lagrangian $\tilde{L}$ of first order in $\theta^{\mu\nu}$ for the gauge and fermion kinetic energy terms are given by

$$\begin{aligned} \tilde{L} = & \left[\frac{1}{4} g^C \theta^{\mu\nu} {\rm Tr}\left(F^C_{\mu\nu} F^C_{\alpha\beta} F^{C\alpha\beta} - 4 F^C_{\alpha\mu} F^C_{\beta\nu} F^{C\alpha\beta} \right) \right. \\ & \left. + (C \to {\rm L}) + (C \to {\rm R}) \right] \\ & - \left[\frac{\rm i}{4} \theta^{\mu\nu} {\rm Tr}\left(\bar{\psi}^{\rm LR} F^{\rm L}_{\mu\nu} \gamma^\alpha D_\alpha \psi^{\rm LR} + F^{\rm R}_{\mu\nu} \bar{\psi}^{\rm LR} \gamma^\alpha D_\alpha \psi^{\rm LR} \right. \right. \\ & \left. + 2 \bar{\psi}^{\rm LR} F^{\rm L}_{\alpha\mu} \gamma^\alpha D_\nu \psi^{\rm LR} + 2 F^{\rm R}_{\alpha\mu} \bar{\psi}^{\rm LR} \gamma^\alpha D_\nu \psi^{\rm LR} \right) \\ & \left. + ({\rm LR} \to {\rm L}C) + ({\rm LR} \to C{\rm R}) \right], \end{aligned} \tag{14}$$

where $D_\mu \psi^{\rm LR} = \partial_\mu \psi^{\rm LR} - {\rm i} g_{\rm L} A^{\rm L}_\mu \psi^{\rm LR} + {\rm i} g_{\rm R} \phi^{\rm LR} A^{\rm R}_\mu$. The above Lagrangian uniquely determines the interactions due to the non-commutative space-time correction to the first order in $\theta^{\mu\nu}$ without the problems pointed out earlier. We emphasize that although the resulting theory at low energies appears to have $U(1)$ factor group(s), the corresponding gauge self-interactions are fixed because of the choice of the trinification group which dictates how gauge bosons interact. From the above Lagrangian one can easily study new interactions due to a non-commutative space-time and test the model by experimental data. For illustration, we present the neutral gauge boson self-interactions and its interactions with the SM fermions. Expanding the above Lagrangian we obtain

$$\begin{aligned} L_{\rm int} = & \frac{1}{4} g^C \theta^{\mu\nu} {\rm Tr}\left(G_{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} - 4 G_{\alpha\mu} G_{\beta\nu} G^{\alpha\beta} \right) \\ & + \frac{1}{16} \theta^{\mu\nu} g_Y \\ & \times \left[c_{\rm W} \left(\frac{7}{15} c^2_{\rm W} + s^2_{\rm W} \right) \left(F_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - 4 F_{\alpha\mu} F_{\beta\nu} F^{\alpha\beta} \right) \right. \\ & - s_{\rm W} \left(\frac{7}{15} s^2_{\rm W} + c^2_{\rm W} \right) \left(Z_{\mu\nu} Z_{\alpha\beta} Z^{\alpha\beta} - 4 Z_{\alpha\mu} Z_{\beta\nu} Z^{\alpha\beta} \right) \\ & + c_{\rm W} \left(c^2_{\rm W} - \frac{23}{15} s^2_{\rm W} \right) \left(F_{\mu\nu} Z_{\alpha\beta} Z^{\alpha\beta} + 2 Z_{\mu\nu} Z_{\alpha\beta} F^{\alpha\beta} \right. \end{aligned}$$

$$
\begin{aligned}
&- 4\left(Z_{\alpha\mu}Z_{\beta\nu}F^{\alpha\beta} + 2F_{\alpha\mu}Z_{\beta\nu}Z^{\alpha\beta}\right)\big) \\
&- s_{\mathrm{W}}\left(s_{\mathrm{W}}^2 - \frac{23}{15}c_{\mathrm{W}}^2\right)\big(Z_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + 2F_{\mu\nu}Z_{\alpha\beta}F^{\alpha\beta} \\
&- 4(F_{\alpha\mu}F_{\beta\nu}Z^{\alpha\beta} + 2Z_{\alpha\mu}F_{\beta\nu}F^{\alpha\beta})\big)\Bigg], \qquad (15)
\end{aligned}
$$

where $c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, $s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$. $G_{\mu\nu}$, $F_{\mu\nu}$, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ are the field strengths for the gluon, photon and Z particles, respectively. Note that the above interactions are obtained at the unification scale where $g_Y = \sqrt{3/5}g^U$ and $\sin^2\theta_{\mathrm{W}} = 3/8$, and $g^C = g^{\mathrm{L}} = g^{\mathrm{R}} = g^U$. From the above we see that the triple neutral gauge boson interactions are uniquely determined unlike the case with the SM gauge group studied in [8]. These interactions are also different from those predicted by the $SU(5)$ model [9]. This can be used to test the model [11]. The fermion–gauge boson interactions can readily be obtained by expanding (14). The Yukawa coupling terms and Higgs potential terms can also be obtained using the results in (6) and (13). We have constructed a NCQFT unification model of the strong and electroweak interactions based on the $SU(3)_C \times SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times Z_3$ group. In this model all interactions are determined. New gauge boson self- and fermion–gauge interactions are predicted. If the non-commutative scale turns out to be low, the model can be tested experimentally. This model provides an example which can consistently describe the strong and electroweak interactions and their unification, and allows a systematic investigation of the hypothesis of non-commutative space-time to be made.

Before closing we would like to make two comments on some possible extensions of the model discussed here. One of them concerns the E_6 extension of the model. The $SU(3)_C \times SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}$ group can be embedded into the E_6 group. One therefore can try to construct a NCQFT based on E_6. With this group the gauge bosons are in the 78 adjoint representation and the fermions and Higgs bosons are in the 27 fundamental representations [12]. A NCQFT model can be constructed following the procedures discussed earlier. This model is very similar to the trinification model with the advantage that no additional Z_3 symmetry is needed. There are however differences and complications. In addition to the gauge bosons in the trinification model, there are also 54 colored gauge bosons. These particles mediate proton decays. Therefore they have to be made heavy. To achieve this more Higgs representations will have to be introduced which complicate the theory [12]. The trinification model is simpler in terms of particle contents.

The other comment concerns another approach to construct the trinification model with non-commutative space-time without the use of the Seiberg–Witten mapping adopted in [13]. In this approach one first constructs fields in $U(N)$ product groups and then breaks the symmetry spontaneously to the $SU(N)$ product group. For the trinification model, one can extend the group to $U(3)_C \times U(3)_{\mathrm{L}} \times U(3)_{\mathrm{R}}$. The gauge field representation is

$$27 = (9,1,1) + (1,9,1) + (1,1,9).$$

In [13] the symmetry breaking of $U(N)$ to $SU(N)$ is assumed to be achieved by non-zero VEVs of representations S_i which transform as singlets under the $SU(N)$ but with non-zero charge for the $U(1)$ subgroup of $U(N)$. Following [13] one can introduce three S_i fields for each of the factor $U(N)$ groups. The VEVs of these fields break the group to the trinification group discussed earlier, producing three heavy gauge bosons. However it has been shown that models based on such an approach violate unitarity [17]. This approach may not lead to a realistic model.

Acknowledgements. This work was supported in part by National Science Council under grants NSC 89-2112-M-002-058 and NSC 89-2112-M-002-065, and in part by the Ministry of Education Academic Excellence Project 89-N-FA01-1-4-3. I would like to thank for hospitality provided by the Institute of Theoretical Sciences at the University of Oregon where part of this work was done.

Note added. Another consistent non-commutative grand unified model based on $SO(10)$ has been constructed by Aschieri et al. (hep-th/0205214), three months after this work.

References

1. H. Snyder, Phys. Rev. **71**, 38 (1947); **72**, 68 (1947); A. Connes, Non-commutative geometry (Academic Press 1994); M. Douglas, N. Nekrasov, hep-th/0106048
2. J. Hewett, F. Petriello, T. Rizzo, Phys. Rev. D **64**, 075012 (2001); P. Mathews, Phys. Rev. D **63**, 075007 (2001); S. Baek et al., Phys. Rev. D **64**, 056001 (2001); H. Grosse, Y. Liao, Phys. Rev. D **64**, 115007 (2001); S. Godfrey, M. Doncheski, Phys. Rev. D **65**, 015005 (2002)
3. M. Hayakawa, Phys. Lett. B **478**, 394 (2000); K. Matsubara, Phys. Lett. B **482**, 417 (2000)
4. A. Armoni, Nucl. Phys. B **593**, 229 (2001)
5. J. Seiberg, E. Witten, JHEP **9909**, 032 (1999)
6. J. Madore et al., Eur. Phys. J. C **16**, 161 (2000); B. Jurco et al., Eur. Phys. J. C **17**, 521 (2000); B. Jurco.P. Schupp, J. Wess, Nucl. Phys. B **604**, 148 (2001); B. Jurco et al., Eur. Phys. J. C **21**, 383 (2001)
7. C. Carlson, C. Carone, R. Lebed, Phys. Lett. B **518**, 201 (2001)
8. X. Calmet et al., Eur. Phys. J. C **23**, 363 (2002)
9. N. Deshpande, X.-G. He, Phys. Lett. B **533**, 116 (2002)
10. P.-M. Ho, H.-C. Kao, Phys. Rev. Lett. **88**, 151602 (2002)
11. W. Behr et al., hep-ph/0202121
12. G. Gursey, P. Ramond, P. Sikive, Phys. Lett. **60**, 177 (1976); Y. Achiman, B. Stech, Phys. Lett. **77**, 389 (1978)
13. M. Chaichian et al., hep-th/0107055
14. M. Chaichian et al., hep-th/0107037
15. A. de Rujula, H. Georgi, S. Glashow, in Fifth Workshop on Grand Unification, edited by K. Kang, H. Fred, P. Frampton, (World Scientific, Singapore 1984), p. 88; K. Babu, X.-G. He, S. Pakvasa, Phys. Rev. D **33**, 763 (1986)
16. S. Willenbrock, hep-ph/0302168
17. J. Hewett, F. Petriello, T. Rizzo, Phys. Rev. D **66**, 036001 (2002)